Complex Numbers

→ Any no. $a + bi$ is a complex number
$a, b \in R$
$i = \sqrt{-1}$

$a + bi$ — real, imaginary

written as : $Z = a + bi$
$W = p + qi$

Z and W are whole numbers

$Re\ Z = a$
real number → real part

$Im\ Z = b$
imaginary numbe → imaginary part

⇒ Operations on complex number

$$Z + W = (a + bi) + (p + qi)$$
$$= (a+p) + i(b+q)$$

$$Z - W = (a+bi) - (p+qi)$$
$$(a-p) + i(b-q)$$

$$Z \times W = (5-2i)\ (1+i)$$
$$= 5 + 5i - 2i - 2i^2$$
$$= 5 + 3i - 2(-1)$$
$$= 5 + 3i + 2$$
$$= 7 + 3i$$

$Re\ (Z \times W) = 7$
$Im\ (Z \times W) = 3$

$$\frac{Z}{W} = \frac{5-2i}{1+i}$$

convert denominator to rational no.
$W = 1 + i$ complex
$W^* = 1 - i$ → Conjugate of w
multiply $\frac{Z}{W}$ with $\frac{W^*}{W^*}$

$$\frac{z}{w} = \frac{5-2i}{1+i}$$

$$w^* = 1-i$$

$$\therefore \frac{z}{w} \times \frac{w^*}{w^*}$$

$$= \frac{5-2i}{1+i} \times \frac{1-i}{1-i} = \frac{5-5i-2i+2i^2}{1-i^2}$$

$$= 5-7i+2(-1)$$

$$= \frac{3-7i}{1-(-1)} = \frac{3-7i}{2} = \frac{3}{2} - \frac{7}{2}i$$

$$Re(z/w) = \frac{3}{2}$$

$$Im(z/w) = \frac{7}{2}$$

⇒ When are 2 complex no. equal

when real parts and imaginary parts are equal.

Q. $2 + xi + y + 2yi = x + 3i$

$2+y + i(x+2y) = x+3i$

$2+y = x$ $\qquad$ $\cancel{i}(x+2y) = 3\cancel{i}$

$2+y = x$ $\qquad$ $x = 3-2y$

$2+y = 3-2y$

$y+2y = 3-2$

$3y = 1$

$\boxed{y = \frac{1}{3}}$

$2 + \frac{1}{3} = x$

$\frac{7}{3} = x$

=> Finding square root of a complex number

☆ square root complex no. is complex no.

Q. find square root of $8+8i$

$$\sqrt{8+8i} = a+bi$$

sq both sides

$$8+8i = (a+bi)^2$$
$$8+8i = a^2+2abi+b^2i^2$$
$$8+8i = (a^2-b^2)+2abi$$

$8 = a^2-b^2$ | $8i = 2abi$

$\boxed{\frac{4}{a} = b}$

$$a^2-b^2 = 8$$
$$a^2-\left[\frac{4}{a}\right]^2 = 8$$
$$\left(a^2-\frac{16}{a^2} = 8\right)a^2$$
$$a^4-16 = 8a^2$$
$$(a^4-8a^2)-16 = 0$$
$$(a^2-4)^2 = 0$$
$$a^2 = 4$$
$$\boxed{a = \pm 2}$$

$b = \frac{4}{a}$

$b = 2$ or $b = -2$

$(2+2i)$ or $(-2-2i)$

arg = angle made by complex number with +ve direction of x-axis.

⇒ Polar form

$$z = a + bi$$

polar co-ordinates

$$z = r(\cos\theta + i\sin\theta) = r\,\mathrm{cis}\,\theta$$

$$|z| = r = \sqrt{a^2+b^2}$$

$$\arg z = \theta = \tan^{-1}\left[\frac{b}{a}\right]$$

$$-\pi \leq \arg z \leq \pi$$

Q. $z = 2 - i$

convert to $z = r(\cos\theta + i\sin\theta)$

$$r = |z| = \sqrt{a^2+b^2} = \sqrt{2^2+1^2} = \sqrt{5}$$

$$\arg z = \theta = -\tan^{-1}\left[\frac{1}{2}\right]$$

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$$= -0.464$$

$$2 - i = \sqrt{5}\left(\cos(-0.464) + i\sin(-0.464)\right)$$

⇒ $z = r\,\mathrm{cis}\,\theta$

$$r = \sqrt{a^2+b^2}$$

$$-\pi \leq \theta \leq \pi$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$|z \times w| = |z|\,|w|$$

$$\arg(z \times w) = \arg z \times \arg w + 2k\pi$$

$$\boxed{\begin{aligned} k &= 0 \\ k &= -1 \\ k &= 1 \end{aligned}}$$

$$\left|\frac{z}{\omega}\right| = \frac{|z|}{|\omega|}$$

$$\arg\left(\frac{z}{\omega}\right) = \arg z - \arg \omega + 2k\pi$$

$$z = r(\cos\theta + i\sin\theta)$$

$$\sqrt{z} = \sqrt{r} \times (\cos\theta + i\sin\theta)^{1/2}$$

★ $(\cos\theta + i\sin\theta)^n$

$= \cos n\theta + i\sin n\theta$

⇒ exponential

$$e^{i\theta} = \text{cis}\,\theta$$

$$z = re^{i\theta}$$

$$e^{i\pi} = \text{cis}\,\theta$$

$$= \cos\pi + i\sin\pi$$

$$= -1 + 0i = -1 + (0 \times i)$$

⇒ Cube root unity.

$$\omega^3 = 1$$

$$\omega = \sqrt[3]{1}$$

$$(\omega^3 - 1) = 0$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$= (\omega - 1)(\omega^2 + \omega + 1) = 0$$

⇒ Locus

1] $|z-u| = r$ → circle with centre u, radius r

2] $|z-u| = |z-w|$ → $\perp^r$ bisector of line UW.

3] $\arg(z-u) = \theta$ → line with angle θ w.r.t x-axis from u